Thermal instability of two-dimensional stagnation point flow with opposing vertical thermal and solutal gradients

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Abstract:

The objective of the present paper is to examine the effects of mass transfer on the thermal instability of a two-dimensional stagnation point flow. We furthermore restrict our attention to thermal and concentration buoyancy forces which are of opposite sign and equal in magnitude. The basic flow has been obtained by solving the governing equations of continuity, momentum, energy and concentration using similarity analysis which are solved numerically using the fourth-order Runge-Kutta method with shooting technique. The stability of the basic flow is then investigated in the usual fashion by making use of the normal mode decomposition. The resulting eigenvalue problem is solved numerically by means of a pseudo spectral collocation method based upon Laguerre's functions. It is found through the calculation of neutral stability curves that concentration buoyancy parameters acts to increase the stability of the basic flow, but Lewis number acts to either increase or decrease it. Increasing Lewis number acts to increase the stability of the basic flow for Le<1, or to decrease it for Le>1. For Le=1 the basic flow is always stable.

Résumé:

L'objectif de la présente étude est d'examiner les effets de transfert de masse sur l'instabilité thermique d'un écoulement stationnaire de couche limite bidimensionnel au point de stagnation. En outre, nous limitons notre attention aux forces de flottabilité thermique et de concentration qui sont de signe opposé et de grandeur égale. L'écoulement de base a été obtenu en résolvant les équations de continuité, de quantité de mouvement, d'énergie et de concentration en utilisant une analyse de similarité, qui sont résolues numériquement en utilisant la méthode de Runge-Kutta d'ordre quatre avec la technique de tir. La stabilité de l'écoulement de base est alors étudiée en se servant de la décomposition en modes normaux. Le problème aux valeurs propres résultant est résolu numériquement par la méthode pseudo-spectrale de Laguerre. Il se trouve par le calcul des courbes de stabilité neutre que les effets de concentration stabilisent l'écoulement de base, tandis que le nombre de Lewis stabilise ou déstabilise. L'augmentation du nombre de Lewis augmente la stabilité de l'écoulement de base pour Le <1, elle diminue pour Le> 1. Pour Le = 1, l'écoulement de base est toujours stable.

Keywords: **Thermal instability, Solutal gradients, Stagnation point, Laguerre's polynomials**

1 Introduction

Double-diffusive convection is formed due to the combination of temperature and concentration gradients in the fluid, in which the thermal and mass diffusivities are different. Thus, the heat and mass transfer occur simultaneously. A substantial amount of research has been reported on double-diffusive convection in confined spaces due to its vast engineering applications. In practice, double diffusive convection may appear in a wide range of scientific fields such as Biology, Astrophysics, Geology, Chemical processes and Crystal-growth techniques [1]. Stagnation flows are found in many applications such as flows over the tips of rockets, aircrafts, submarines and oil ships. The study of the stagnation flow problem was started by Hiemenz [2] who developed an exact solution to the Navier-Stokes governing equations for the forced convection case. The theory of stagnation region with heat and mass transfer to fluid jets impinging normally on solid surfaces is discussed by Kendoush [3]. Similarity solutions for hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media has been presented by Chamkha and Khaled [4]. Takhar et al [5] studied the magnetohydrodynamic (MHD) flow with mass transfer in a viscous fluid bounded by a stretching surface with non-zero slot velocity. Hydromagnetic flow over a stretching sheet in the presence of heat and mass transfer is presented by Liu [6]. Seddeek et al [7] studied Effect of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media with radiation. Tsai and Huang [8] investigated the Heat and mass transfer for Soret and Dufour's effects on Hiemenz flow through porous medium onto a stretching surface. Heat and mass transfer for Soret and Dufour's effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid has been analyzed by Hayat et al [9].

The first studies [10,11,12] of the stability of Hiemenz flow, reveal that it is always stable to three dimensional self-similar disturbances. Lyell and Huerre [13] examined the planar stagnation flow problem by using Galerkin expansion method and discussed the linear and non linear stability. Hall et al. [14] extended the work of Wilson and Gladwell [12] to include the effects of crossflow in the freestream and suction or blowing at the wall. Brattkus and Davis [15] considered a wider class of disturbances for the stability of the incompressible Hiemenz flow and concluded that these modes were more stable compared to the self-similar disturbances. When buoyancy alone is taken into account, computations of Chen et al. [16] revealed that thermal excitation generates instabilities when the Rayleigh number exceeds some critical value. Amaouche et al. [17] studied the effect of a constant magnetic field on the thermal instability of a two dimensional stagnation point flow and they found that magnetic field act to increases its stability.

It is the purpose of this paper to explore the effects of mass transfer on the thermal instability of a two dimensional stagnation point flow. We furthermore restrict our attention to thermal and concentration buoyancy forces which are of opposite sign and equal in magnitude. It will be shown that these mechanisms act to oppose each other; thermal buoyancy is to destabilize the flow whereas concentration buoyancy effects are to increase its stability.

2 Analysis

2.1 Governing equations

We consider a steady laminar two-dimensional flow impinging normally on a static horizontal flat plate, in a viscous fluid of temperature T_∞ and concentration C_∞ is investigated. The flat plate is in the (x^*, z^*) plane and the basic flow is two-dimensional in the (x^*, y^*) plane, the x^* axis being the streamwise direction along the plate, and the z^* axis the spanwise direction. The stagnation streamline coincides with the y^* axis. The external velocity is prescribed as $\mathbf{v}_e^*(a\,x^*,-a\,y^*,0)$; where a is constant ($a \geq 0$). T_w and C_w are temperature and concentration at the wall assumed to be constant, where $T_w > T_\infty$ and $C_w < C_\infty$. The buoyancy forces arise due to the variations in temperature and concentration of fluid. The Boussinesq approximation is invoked for the fluid properties to relate the density changes to temperature and concentration and to couple in this way the temperature and concentration fields to the flow field. Under these assumptions, the equations of continuity, momentum, energy and concentration are cast in their dimensional form in terms of the temperature T, concentration C and velocity $\mathbf{v}^*$ as follow:

$$\nabla \cdot \mathbf{v}^* = 0 \tag{1}$$

$$\frac{\partial \mathbf{v}^*}{\partial t^*} + \left(\mathbf{v}^* \cdot \nabla\right)\mathbf{v}^* = -\frac{1}{\rho}\nabla p^* + \nu \nabla^2 \mathbf{v}^* - g\beta_T\left(T - T_\infty\right) - g\beta_s\left(C - C_\infty\right) \tag{2}$$

$$\frac{\partial T}{\partial t^*} + \left(\mathbf{v}^* \cdot \nabla\right)T = \alpha \nabla^2 T \tag{3}$$

$$\frac{\partial C}{\partial t^*} + \left(\mathbf{v}^* \cdot \nabla\right)C = D_m \nabla^2 C \tag{4}$$

With boundary conditions:

$$\mathbf{v}^*\left(x^*,0\right) = 0, T\left(x^*,0\right) = T_w, C\left(x^*,0\right) = C_w, \mathbf{v}^*\left(x^*,\infty\right) = \mathbf{v}_e^*\left(x^*,y^*\right), T\left(x^*,\infty\right) = T_\infty, C\left(x^*,\infty\right) = C_\infty \tag{5}$$

In the above equations $\mathbf{v}^*$ is the velocity vector of the fluid, t the time, p^* the pressure. ρ and ν are the fluid density and kinematic viscosity respectively. $\mathbf{g}$ is the gravitational acceleration, β_T and, β_s are the expansion coefficients of temperature and concentration respectively. α, D_m are the thermal and mass diffusivity respectively, T and C are the temperature and concentration respectively. The subscripts w and ∞ stand respectively for the wall and free stream conditions, asterisks indicate dimensional quantities.

2.2 Solution of the basic flow

By using stream function formalism to resolve the basic steady and two-dimensional flow, Eqs (1–3) can be rewritten, after eliminating the pressure, as follow:

$$\psi_x\left(\nabla^2\psi\right)_x - \psi_x\left(\nabla^2\psi\right)_y - \nabla^2\left(\nabla^2\psi\right) + Gr\left(\theta_x + Nc\phi_x\right) = 0 \qquad (6)$$

$$\nabla^2\theta + Pr\left(\psi_x\theta_y - \psi_y\theta_x\right) = 0 \qquad (7)$$

$$\nabla^2\phi + Le\,Pr\left(\psi_x\phi_y - \psi_y\phi_x\right) = 0 \qquad (8)$$

The boundary conditions become:

$$\psi_y(x,0) = 0, \psi_x(x,0) = 0, \theta(x,0) = 1, \phi(x,0) = 1, \psi_y(x,\infty) = x, \theta(x,\infty) = 0, \phi(x,\infty) = 0 \qquad (9)$$

The above equations are, for convenience, expressed in terms of dimensionless variables. Distances and time are scaled using the factors $\ell = (\nu/a)^{1/2}$ and a^{-1}, respectively [17]. The stream function ψ is referred to ν , the scaled temperature and concentration are defined by $\theta(x,y)=(T-T_\infty)/(T_w-T_\infty)$, $\phi(x,y)=(C-C_\infty)/(C_w-C_\infty)$ respectively. The parameters in the problem are Prandtl number $Pr=\nu/\alpha$, Lewis number $Le=\alpha/D_m$ and Grashof number $Gr=g\beta_T(T_w-T_\infty)\ell^3/\nu^2$. Here, the buoyancy ratio parameter $Nc=\beta_s(C_w-C_\infty)/(\beta_T(T_w-T_\infty))$ measures the relative importance of mass and thermal diffusion in the buoyancy driven flow. Nc will be zero for purely thermal buoyancy driven flow, infinite for mass driven flow, positive for aiding the flow, and negative for opposing the flow, in the present paper we will consider only the case in which the buoyancy forces due to the thermal and concentration gradients are opposite and equal ($Nc=-1$).

After introducing the similarity transformation:

$$\psi(x,y) = xf(y), \theta(x,y) = \theta(y), \phi(x,y) = \phi(y) \qquad (10)$$

the equations of (6-8) can be written over in terms of $f(y)$, $\theta(y)$ and $\phi(y)$:

$$f''' + ff'' + 1 - f'^2 = 0 \qquad (11)$$

$$\theta'' + Pr\, f\theta' = 0 \qquad (12)$$

$$\phi'' + Le\,Pr\, f\phi' = 0 \qquad (13)$$

where the prime denotes a partial differentiation with respect to y. The transformed boundary conditions are given by

$$f(0) = 0, f'(0) = 0, \theta(0) = 1, \phi(0) = 1, f'(\infty) = 1, \theta(\infty) = 0, \phi(\infty) = 0 \qquad (14)$$

The nonlinear differential equations (11–13) along with the boundary conditions (14) are solved numerically using the fourth-order Runge-Kutta method with shooting technique.

2.3 Linear stability analysis

In order to study the linear stability of the above basic flow we consider infinitesimally small disturbances propagating along the boundary layer, so that the instantaneous quantities $\bar{u}, \bar{v}, \bar{w}, \bar{p}$ and $\bar{\theta}$ can be expressed as:

$$\left(\bar{u}, \bar{v}, \bar{w}, \bar{p}, \bar{\theta}, \bar{\phi}\right)(x,y,z,t) = \left(u, v, 0, p, \theta, \phi\right)(x,y) + \left(\tilde{u}, \tilde{v}, \tilde{w}, \tilde{p}, \tilde{\theta}, \tilde{\phi}\right)(x,y,z,t) \qquad (15)$$

Where $(u, v, 0, p, \theta, \phi)$ and $\left(\tilde{u}, \tilde{v}, \tilde{w}, \tilde{p}, \tilde{\theta}, \tilde{\phi}\right)$ represent basic-state and disturbance-state quantities, respectively. Substituting the above expression into the governing equations of continuity, momentum and energy, subtracting the basic state, and linearizing with respect to the small perturbations gives a set of linearized perturbation equations:

$$\tilde{u}_x + \tilde{v}_y + \tilde{w}_z = 0 \tag{16}$$

$$\tilde{u}_t + u\tilde{u}_x + v\tilde{u}_y + u_x\tilde{u} + u_y\tilde{v} + \tilde{p}_x - \left(\tilde{u}_{xx} + \tilde{u}_{yy} + \tilde{u}_{zz}\right) = 0 \tag{17}$$

$$\tilde{v}_t + u\tilde{v}_x + v\tilde{v}_y + v_x\tilde{u} + v_y\tilde{v} + \tilde{p}_y - \left(\tilde{v}_{xx} + \tilde{v}_{yy} + \tilde{v}_{zz}\right) - Gr\left(\tilde{\theta} - \tilde{\phi}\right) = 0 \tag{18}$$

$$\tilde{w}_t + u\tilde{w}_x + v\tilde{w}_y + \tilde{p}_z - \left(\tilde{w}_{xx} + \tilde{w}_{yy} + \tilde{w}_{zz}\right) = 0 \tag{19}$$

$$\tilde{\theta}_t + u\tilde{\theta}_x + v\tilde{\theta}_y + \theta_x\tilde{u} + \theta_y\tilde{v} - \left(1/Pr\right)\left(\tilde{\theta}_{xx} + \tilde{\theta}_{yy} + \tilde{\theta}_{zz}\right) = 0 \tag{20}$$

$$\tilde{\phi}_t + u\tilde{\phi}_x + v\tilde{\phi}_y + \phi_x\tilde{u} + \phi_y\tilde{v} - \left(1/\left(Le\,Pr\right)\right)\left(\tilde{\phi}_{xx} + \tilde{\phi}_{yy} + \tilde{\phi}_{zz}\right) = 0 \tag{21}$$

Hence, the problem consists of a set of coupled six-dimensional partial-differential equations with variable coefficients. These coefficients, which depend on the basic flow, change strongly in the normal direction and linearly in the chordwise direction, but not in the spanwise direction. As a consequence, the solution is separable in the variables *z* and *t*, and retaining the self similarity for the perturbation amplitude, the disturbance quantities of a general travelling mode can be expressed in the form

$$\left(\tilde{u},\tilde{v},\tilde{w},\tilde{p},\tilde{\theta},\tilde{\phi}\right)\left(x,y,z,t\right) = \left(x\hat{u},\hat{v},\hat{w},\hat{p},\hat{\theta},\hat{\phi}\right)\left(y\right)exp\left(ikz + \omega t\right) \tag{22}$$

Where *k* denotes the spanwise wavenumber and ω the temporal growth rate and $x\hat{u},\hat{v},\hat{w},\hat{p},\hat{\theta},\hat{\phi}$ are complex amplitude functions of three-dimensional small disturbances. Consequently, the stability results are limited to the special class of self similar disturbances considered above. Substituting the decomposition (22) into the equations (16–21), one obtains

$$\hat{u} + D\hat{v} + ik\hat{w} = 0 \tag{23}$$

$$\left(D^2 + fD - 2f' - k^2\right)\hat{u} - f''\hat{v} = \omega\hat{u} \tag{24}$$

$$\left(D^2 + fD + f' - k^2\right)\hat{v} - D\hat{p} + Gr\left(\hat{\theta} - \hat{\phi}\right) = \omega\hat{v} \tag{25}$$

$$\left(D^2 + fD - k^2\right)\hat{w} - ik\hat{p} = \omega\hat{w} \tag{26}$$

$$-Pr\,\theta'\hat{v} + \left(D^2 + Pr\ fD - k^2\right)\hat{\theta} = Pr\,\omega\hat{\theta} \tag{27}$$

$$-Le\,Pr\,\phi'\hat{v} + \left(D^2 + Le\,Pr\ fD - k^2\right)\hat{\phi} = Le\,Pr\,\omega\hat{\phi} \tag{28}$$

Here $D^n \equiv d^n/dy^n$. As boundary conditions, zero perturbation is imposed on $\hat{u},\hat{v},\hat{w}$ and zero derivative for the normal perturbation velocity, when temperature, concentration and pressure perturbations are assumed to vanish at the wall

$\hat{u} = \hat{v} = \hat{w} = \hat{p} = \hat{\theta} = \hat{\phi} = D\hat{v} = 0$ as $y = 0$

In the far-field, condition of vanishing perturbations at a sufficiently distance from the wall may be imposed

$\hat{u} = \hat{v} = \hat{w} = \hat{p} = \hat{\theta} = \hat{\phi} = 0$ as $y \to \infty$

This system is to be solved numerically in order to determine the eigenvalue ω along with the corresponding eigenfunctions $\hat{u},\hat{v},\hat{w},\hat{p},\hat{\theta}$ and $\hat{\phi}$ as functions of the spanwise wave number *k* for the three parameters characterizing the overall fluid system and the base state, *Gr*, *Pr* and *Le*. At marginality, corresponding to ω=0, only for some characteristic values of *Gr* will the problem allow a non trivial solution for given *k*, *Pr* and *Le*. In order to approximate the solution of the differential eigenvalue system to be solved and incorporate its exponential damping at infinity, we shall use a pseudo spectral method based on Laguerre's functions expansion, i.e., Laguerre's polynomials $L_n(y)$ multiplied by a decaying exponential. So, truncating the expansion after a finite number *N* of terms, an approximation F_N of a function *F* is thought in the form $\Phi_N(y)exp(-y)$ with Φ_N being a polynomial of degree at most *N* and forced to fulfill the linear system at collocation nodes y_j ($j=1,\ldots N$) which are selected to be the zeroes of $L_N(y)$. After the incorporation of the homogenous condition at $y_0=0$, an approximation of $\Phi_N(y)$ is written in the form [17].

$$\Phi_N\left(y\right) = \sum_{j=1}^{N} \frac{yL_N\left(y\right)}{y_j\left(y - y_j\right)\dfrac{dL_N}{dy}\left(y_j\right)}\Phi_N\left(y_j\right) \tag{29}$$

So, the discretization procedure transforms the differential equations (23–28) into an algebraic eigenvalue problem expressed in terms of discretized square matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ such that

$$\boldsymbol{A}(k,Pr,Le,Gr)\boldsymbol{\Phi}_N = \omega \boldsymbol{B}(k,Pr,Le)\boldsymbol{\Phi}_N \qquad (30)$$

Here, $\boldsymbol{\Phi}_N$ is a vector of expansion coefficients.

3 Results and discussion

The stability results, overall, depict that the onset conditions of thermal instability are significantly affected by the presence of mass transfer. Globally one can observe that mass transfer acts to reduce instability when compared to the case without mass transfer. An overview of the stability properties of the basic flow can be seen from the sequence of neutral stability curves displayed in Figure 1 for a typical value of Pr by varying Le : a) for Le <1, b) for Le >1. The unstable region lies above the curve while the stable region lies below it. First, we can see that the instability region in the (k,Gr) plane is always contained in that corresponding to the case without mass transfer. Results indicate that the influence of Le changes when it is smaller or greater than unity. When $Le<1$ a regular stabilization effect is observed if Le grows, but this effect is inversed when Le >1. This observation is confirmed in a global picture provided by Figure 2 showing the critical Grashof number as functions of Le for two selected values of Prandtl number. One can observe that critical Grashof Gr_c grows rapidly to infinity when Le tends to unity but the change in Gr_c becomes imperceptible to the change in Le as it tends to zero or infinity. This observation can be explained by the influence of the buoyancy term $Gr\left(\tilde{\theta}-\tilde{\phi}\right)$ in equation (18) which is the source of instability. When $Le \rightarrow 1$ this term tends to be neglected and so the destabilization effect disappears and one obtains a stable basic flow. But when Le go away from unity the term grows due to the large difference between temperature and concentration which leads to increase instability.

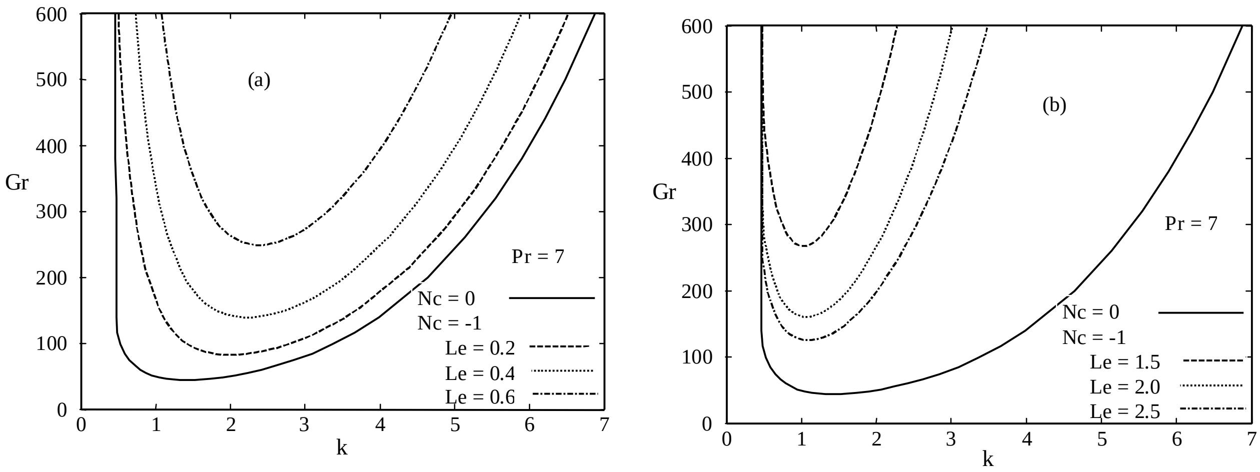

FIG. 1 – Neutral stability curves for different values of Lewis number (a) Le<1, (b) Le>1.

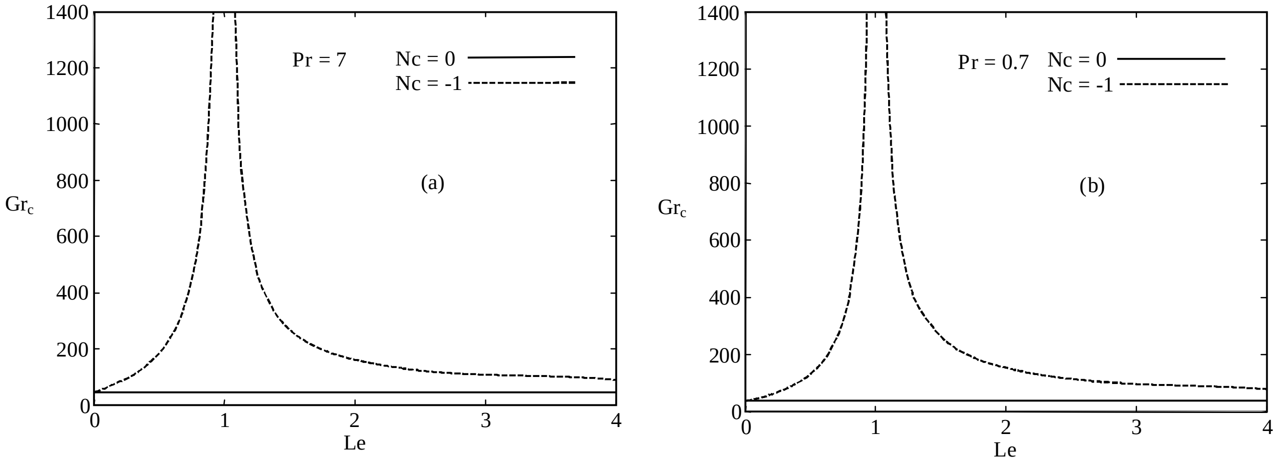

FIG. 2 – Critical Grashof number Gr_c versus the Lewis number (a) $Pr=7$, (b) $Pr=0.7$.

4 Conclusion

Results presented herein about new stability study of two dimensional double diffusive stagnation point flow with opposing temperature and concentration gradients can be summarized as follows. First, it has been found that the presence of mass transfer yields i) more important critical Grashof number for the onset of instability, ii) the reduction of the region of instability in the (*k*,*Gr*) plane, iii) the decrease of the temporal growth rate of the most unstable mode and of the least stable one. In particular, we found that the effect of Lewis number is to retard thermal instability when it is close to unity. But, its effect becomes imperceptible on stability for higher values exactly the same effect of Prandtl number for higher values.

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